Stochastic

31/10/2015 السبت د. سعيد سلام محاضرة 5

* Stochastic (Random) Variables :-

Consider the experiment of rolling a pair of fair dice. Let X denotes the sum rolled. Evedently X is a Stochastic variable which takes the values

$$X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

each with certain probability. For example

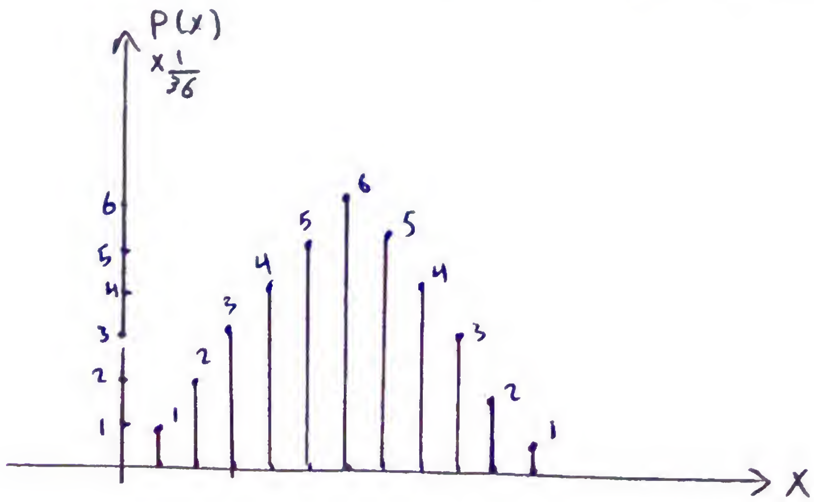
Let E_1 : The sum being 6.

Then $E_1 = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

$$P(E_1) = \left\{ \left(\frac{1}{6} \cdot \frac{1}{6}\right) + \cdots \right\} = \frac{5}{36}$$

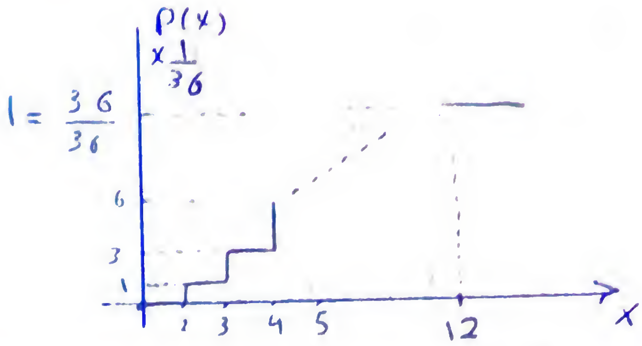
The probabilities of other sums are shown in the table and figure

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36



* Commulative Distribution function

Based on the previous figure:-



It is defined as a function F(x) equals the probability that the stochastic variable X takes values less than or equal to a value x:

$$F(x) = P(X \leq x$$

for example $F(5) = P(X \leq 5)$

The distribution function is an increasing function and has the following properties:-

i- $F(-\infty) = 0$

ii- $F(\infty) = 1$

iii- $F(a) \leq F(b)$ for $a \leq b$

If X is a discrete random variable with probability P, then F(x) is the step function defined by

$$F(x) = \sum_{x_i \leq x} P(x_i)$$

⟹ Turn over

on the other hand, if X is a continuous random Variable with distribution p, then

$$F(X) = \int_{-\infty}^{x} p(t)\,dt$$

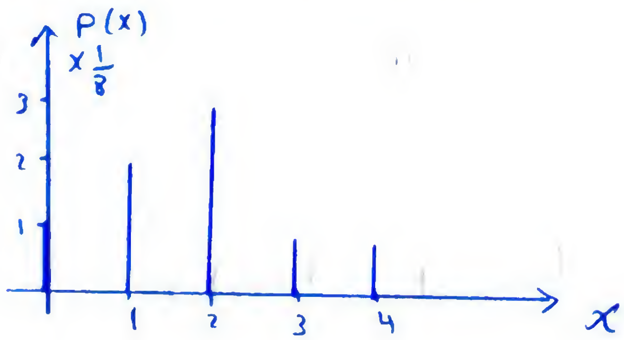
In either cases, F(x) is monotonic increasing

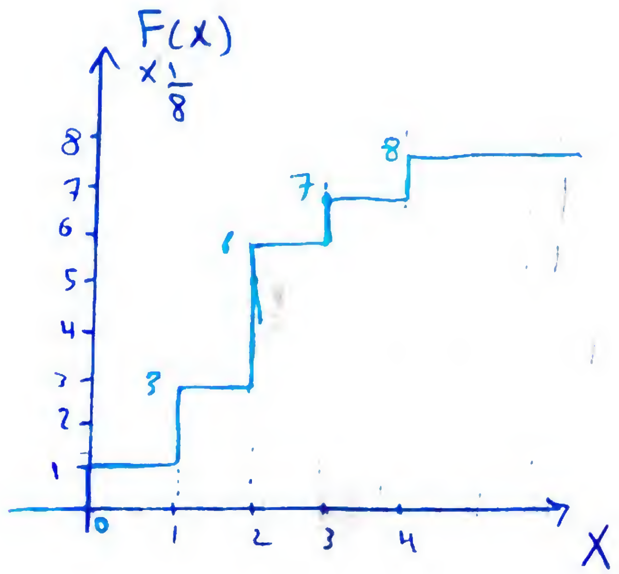
Example:- Let x be a discrete random variable with the probability function

x	0	1	2	3	4
P(x)	1/8	2/8	3/8	1/8	1/8

P(x) = 0 elsewhere, find the comm. dist. fn .

~~F(-1) = F(x ≤ -1) = F(-∞) = 0~~





$F(-1) = F(X \leq -1) = F(-\infty) = 0$

$F(0) = P(X \leq 0) = \frac{1}{8}$, $F(1/2) = P(X \leq 1/2) = 1/8$

$F(1) = P(X \leq 1) = P(0) + P(1) = \frac{1}{8} + \frac{2}{8} = 3/8$

$F(2) = P(X \leq 2) = P(0) + P(1) + P(2) = \dots = 6/8$

$F(3) = P(X \leq 3) = P(0) + P(1) + P(2) + P(3) = 7/8$

$F(4) = P(X \leq 4) = P(0) + P(1) + P(2) + P(3) + P(4) = 1$

$F(5) = F(6) = \dots = F(\infty) = 1$

CK: $F(-\infty) = 0$, $F(\infty) = 1$, $F(1) < F(2)$, $F(1.5) = F(1)$

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* Density function : (probability Density fn)
(Prob. fn)
(Density fn)

If $F(x)$ is a continuous function and if $f(x) = \frac{dF(x)}{dx}$, then $f(x)$ is called the density function.

$$\int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{x} dF(x) = F(x)\Big|_{-\infty}^{x} = F(x) - \underbrace{F(-\infty)}_{0} = F(x)$$

- Properties of the density function:-

i - $f(x) \geq 0$, hence $F(x)$ is an increasing function, then $\frac{dF}{dx} \geq 0$

ii - $\int_{-\infty}^{\infty} F(x)\,dx = 1$, hence

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} dF(x) = F(x)\Big|_{-\infty}^{\infty} = F(\infty) - \underbrace{F(-\infty)}_{0} = 1$$

The area under the density curve always equals one

<u>Note</u>: If x is discrete, then

$$\sum_{-\infty}^{\infty} f(x) = 1$$

example:-

Let $f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ \frac{3-x}{4} & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$

- Is $f(x)$ a density function?
- If so, find the distribution $\underline{fn}$ $F(x)$